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Bipolar Hypersoft Sets

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Abstract: Hypersoft set theory is an extension of soft set theory and is a new mathematical tool for dealing with fuzzy problems; however, it still suffers from the parametric tools' inadequacies. In order to boost decision-making accuracy even more, a new mixed mathematical model called the bipolar hypersoft set is created by merging hypersoft sets and bipolarity. It is characterized by two hypersoft sets, one of which provides positive information and the other provides negative information. Moreover, some fundamental properties relative to it such as subset, superset, equal set, complement, difference, relative (absolute) null set and relative (absolute) whole set are defined. Furthermore, some set-theoretic operations such as the extended intersection, the restricted union, intersection, union, AND-operation and OR-operation of two bipolar hypersoft sets with their properties are discussed and supported by examples. Finally, tabular representations for the purposes of storing bipolar hypersoft sets in computer memory are used.

Keywords: bipolar hypersoft sets; hypersoft sets; bipolar soft sets; restricted union; extended intersection; difference; AND-operation; OR-operation

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1. Introduction

A soft set is made up of two parts—a predicate and an approximate value set—and it provides an approximate description of the object under consideration. In classical mathematics, exact solutions to mathematical models are often needed. If the model's complications or complexities increase, it becomes more difficult to obtain exact solutions; instead, approximate solutions can be obtained by using a variety of methods. In soft set theory, on the other hand, we do not need to incorporate the principle of exact solution since the initial description of the object is approximate.

Soft set theory is a fully generic mathematical tool for modeling uncertainties that was introduced by Russian researcher Molodtsov [1] in 1999. There are no restrictions on how objects are described and, thus, researchers can use any type of parameter they want, which significantly simplifies the decision-making process and renders it more effective and accurate in the face of incomplete data. Many techniques exist for modeling real-world complex systems, including classical probability theory, fuzzy set theory, interval mathematics and so on. All of these techniques have a weak point in that they lack parameterization, which means that they cannot be used to tackle problems in fields such as economics, environmental science and social science. Soft set theory is largely free of the difficulties associated with the methods described above and it has a broader scope for a variety of multidimensional applications. In the sense of soft sets, various operations analogous to union, intersection, complement, difference and others in set theory have been discussed. Definitions and results can be found in [2–5]. Moreover, some important works on soft sets can be found in [6–10].

In 2013, Shabir and Naz [11] independently brought out the concept of bipolar soft set as a hybrid of bipolarity [12] and soft set theory [1]. Our decision-making is based on two

sides, which are positive and negative, according to Dubois and Prade [12] and we choose based on which one is better

A bipolar soft set is provided by considering not only a carefully chosen set of parameters but also an allied set of opposite meaning parameters named "Not set of parameters". As a result, a bipolar soft set is made up of two soft sets, one of which represents the positive side and the other the negative. Some notions, properties, operations and a bipolar soft set application in decision-making problems were investigated in [11]. Later on, Fatimah, Rosadi, Hakim and Alcantud [13] proposed N-soft sets and they solved decision-making algorithms by using N-soft sets. Naz and Shabir [14] brought out the concept of fuzzy bipolar soft sets and discussed their algebraic structures and applications. Abdullah, Aslam and Ullah [15] introduced the bipolar fuzzy soft set and studied its applications in the decision-making problem. Karaaslan and Çağman [16] pointed out the bipolar soft rough set and studied utilization in decision making. Shabir and Gul [17] modified rough bipolar soft sets. Karaaslan and Karatas [18] reintroduced a bipolar soft set by using a bijective map between a set of parameters and its negative. Karaaslan, Ahmad and Ullah [19] constructed bipolar soft groups and investigated some of its properties. Ozturk [20] defined bipolar soft topological spaces and obtained some properties and results on them. Fadel and Dzul-Kifli [21] provided more properties on bipolar soft topological spaces. Malik and Shabir [22] successfully applied rough fuzzy bipolar soft sets in decision-making problems. Many more authors have auspiciously adapted bipolar soft set theory in various fields (see Riaz and Tehrim [23–26], Zhang [27], Ali et al. [28], Al-shami [29], Mahmood [30], Wang et al. [31], Ali et al. [32], Kamacı and Petchimuthu [33] and Lee and Hur [34]).

On the other part, by replacing the function F with a multi-argument function defined on the Cartesian product of n different sets of parameters, Smarandache [35] extended the notion of a soft set to the hypersoft set in 2018. This definition is more adaptable than the soft set and better suited for decision-making problems. A hypersoft set has gained more importance as a generalization of soft sets and has been investigated for possible extensions in many fields of mathematics. Saeed et al. [36,37] and Abbas et al. [38] introduced and studied several operations on hypersoft sets. Recently, hypersoft sets have been expanded by embedding the idea of the fuzzy set, intuitionistic fuzzy set, neutrosophic set and plithogenic set to expand the field of application of hypersoft sets. For instance, the fuzzy hypersoft set [35,39], intuitionistic fuzzy hypersoft set [35,40], neutrosophic hypersoft set [35,41], plithogenic hypersoft set [35], convex hypersoft sets and concave hypersoft sets [42] are some of the focuses of certain studies. Recently, several researchers have auspiciously adapted the hypersoft set theory in various fields (see Martin et al. [43], Zulqarnain et al. [44,45], Ahmad et al. [46], Zhang [27], Deli [47], Al-Tahan et al. [48], Saqlain et al. [49] and Rahman et al. [50]).

The paper is structured as follows: Firstly, we recall the necessary background on hypersoft sets and bipolar soft sets. Secondly, we present some operations on bipolar hypersoft set theory, such as subset, complement, difference, extended intersection, restricted union, intersection, union, difference, AND and OR with their properties. Thirdly, we proposed the necessity and possibility operations and some properties and examples. Furthermore, tabular representations are used for the purpose of storing bipolar hypersoft sets in a computer. Finally, we conclude the paper.

2. Hypersoft Sets and Bipolar Soft Sets

2.1. Hypersoft Sets

Let $\mathcal{U}$ be an initial universe, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$ and $E_1, E_2, ..., E_n$ the pairwise of disjoint sets of parameters. Let $A_i, B_i \subseteq E_i$ for $i = 1, 2, ..., n$.

Definition 1 ([35])**.** *A pair $(\mathbb{F}, A_1 \times A_2 \times ... \times A_n)$ is called a hypersoft set over $\mathcal{U}$, where $\mathbb{F}$ is a mapping given by $\mathbb{F} : A_1 \times A_2 \times ... \times A_n \to \mathcal{P}(\mathcal{U})$.*

For the sake of simplicity, we write the symbol $\mathcal{E}$ for $E_1 \times E_2 \times ... \times E_n$ and for the subsets of $\mathcal{E}$. The symbols $\mathcal{A}$ for $A_1 \times A_2 \times ... \times A_n$, $\mathcal{B}$ for $B_1 \times B_2 \times ... \times B_n$ and $\mathcal{C}$ for $C_1 \times C_2 \times ... \times C_n$. Clearly, each element in $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{E}$ is an n-tuple element.

Definition 2 ([36]). *For two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$, we say that $(\mathbb{F}, \mathcal{A})$ is a hypersoft subset of $(\mathbb{G}, \mathcal{B})$ if the following is the case:*

1. $\mathcal{A} \subseteq \mathcal{B}$ *and;*
2. $\mathbb{F}(\alpha) \subseteq \mathbb{G}(\alpha)$ *for all* $\alpha \in \mathcal{A}$.

We write $(\mathbb{F}, \mathcal{A}) \,\widetilde{\subseteq}\, (\mathbb{G}, \mathcal{B})$.

$(\mathbb{F}, \mathcal{A})$ is said to be a hypersoft superset of $(\mathbb{G}, \mathcal{B})$ if $(\mathbb{G}, \mathcal{B})$ is a hypersoft subset of $(\mathbb{F}, \mathcal{A})$. We denote it by $(\mathbb{F}, \mathcal{A}) \,\widetilde{\supseteq}\, (\mathbb{G}, \mathcal{B})$.

Definition 3 ([36]). *Two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$ are said to be hypersoft equal if $(\mathbb{F}, \mathcal{A})$ is a hypersoft subset of $(\mathbb{G}, \mathcal{B})$ and $(\mathbb{G}, \mathcal{B})$ is a hypersoft subset of $(\mathbb{F}, \mathcal{A})$.*

Definition 4 ([36]). *Let $\mathcal{A} = \{\alpha_1, \alpha_2, ..., \alpha_n\}$ be a set of parameters. The NOT set of $\mathcal{A}$ denoted by $\neg\mathcal{A}$ is defined by $\neg\mathcal{A} = \{\neg\alpha_1, \neg\alpha_2, ..., \neg\alpha_n\}$ where $\neg\alpha_i =$ not α_i for $i = 1, 2, ..., n$.*

The following results are obvious.

Proposition 1. *For any subsets $\mathcal{A}, \mathcal{B} \subseteq \mathcal{E}$.*

1. $\neg(\neg\mathcal{A}) = \mathcal{A}$.
2. $\neg(\mathcal{A} \cup \mathcal{B}) = \neg\mathcal{A} \cup \neg\mathcal{B}$.
3. $\neg(\mathcal{A} \cap \mathcal{B}) = \neg\mathcal{A} \cap \neg\mathcal{B}$.

Definition 5 ([36]). *The relative complement of a hypersoft set $(\mathbb{F}, \mathcal{A})$ is denoted by $(\mathbb{F}, \mathcal{A})^c$ and is defined by $(\mathbb{F}, \mathcal{A})^c = (\mathbb{F}^c, \mathcal{A})$ where $\mathbb{F}^c : \mathcal{A} \to \mathcal{P}(\mathcal{U})$ is a mapping given by $\mathbb{F}^c(\alpha) = \mathcal{U} \setminus \mathbb{F}(\alpha)$ for all $\alpha \in \mathcal{A}$.*

Definition 6 ([37]). *A hypersoft set $(\mathbb{F}, \mathcal{A})$ over $\mathcal{U}$ is said to be a relative null hypersoft set denoted by $(\mathbb{F}, \mathcal{A})_\Phi$ if for all $\alpha \in \mathcal{A}$, $\mathbb{F}(\alpha) = \phi$.*

Definition 7 ([37]). *A hypersoft set $(\mathbb{F}, \mathcal{A})$ over $\mathcal{U}$ is said to be a relative whole hypersoft set denoted by $(\mathbb{F}, \mathcal{A})_\Psi$ if for all $\alpha \in \mathcal{A}$, $\mathbb{F}(\alpha) = \mathcal{U}$.*

Definition 8 ([37]). *A hypersoft set $(\mathbb{F}, \mathcal{A})$ over $\mathcal{U}$ is said to be an absolute whole hypersoft set denoted by $(\mathbb{F}, \mathcal{E})_\Psi$ if for all $\alpha \in \mathcal{E}$, $\mathbb{F}(\alpha) = \mathcal{U}$.*

Definition 9 ([36]). *The union of two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$ is a hypersoft set $(\mathbb{H}, C)$, where $C = \mathcal{A} \cup \mathcal{B}$ and for all $\alpha \in C$.*

$$\mathbb{H}(\alpha) = \begin{cases} \mathbb{F}(\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{G}(\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{F}(\alpha) \cup \mathbb{G}(\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

We write $(\mathbb{F}, \mathcal{A}) \,\widetilde{\cup}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

Definition 10 ([37]). *The extended intersection of two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$ is a hypersoft set $(\mathbb{H}, C)$, where $C = \mathcal{A} \cup \mathcal{B}$ and for all $\alpha \in C$.*

$$\mathbb{H}(\alpha) = \begin{cases} \mathbb{F}(\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{G}(\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{F}(\alpha) \cap \mathbb{G}(\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

We write $(\mathbb{F}, \mathcal{A}) \,\widetilde{\cap}_{\varepsilon}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

Definition 11 ([37]). *The restricted union of two hypersoft sets* $(\mathbb{F}, \mathcal{A})$ *and* $(\mathbb{G}, \mathcal{B})$ *over a common universe* $\mathcal{U}$ *is a hypersoft set* $(\mathbb{H}, C)$, *where* $C = \mathcal{A} \cap \mathcal{B}$ *and for all* $\alpha \in C$, $\mathbb{H}(\alpha) = \mathbb{F}(\alpha) \cup \mathbb{G}(\alpha)$. *We write* $(\mathbb{F}, \mathcal{A}) \,\widetilde{\cup}_{\Re}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

Definition 12 ([36]). *The intersection of two hypersoft sets* $(\mathbb{F}, \mathcal{A})$ *and* $(\mathbb{G}, \mathcal{B})$ *over a common universe* $\mathcal{U}$ *is a hypersoft set* $(\mathbb{H}, C)$, *where* $C = \mathcal{A} \cap \mathcal{B}$ *and for all* $\alpha \in C$, $\mathbb{H}(\alpha) = \mathbb{F}(\alpha) \cap \mathbb{G}(\alpha)$. *We write* $(\mathbb{F}, \mathcal{A}) \,\widetilde{\cap}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

Definition 13 ([37]). *The OR-operation of two hypersoft sets* $(\mathbb{F}, \mathcal{A})$ *and* $(\mathbb{G}, \mathcal{B})$ *over a common universe* $\mathcal{U}$ *is a hypersoft set* $(\mathbb{H}, C)$, *where* $C = \mathcal{A} \times \mathcal{B}$ *and for* $(\alpha, \beta) \in C$, $\alpha \in \mathcal{A}$, $\beta \in \mathcal{B}$, $\mathbb{H}(\alpha, \beta) = \mathbb{F}(\alpha) \cup \mathbb{G}(\beta)$. *We write* $(\mathbb{F}, \mathcal{A}) \,\widetilde{\vee}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

Definition 14 ([37]). *The AND-operation of two hypersoft sets* $(\mathbb{F}, \mathcal{A})$ *and* $(\mathbb{G}, \mathcal{B})$ *over a common universe* $\mathcal{U}$ *is a hypersoft set* $(\mathbb{H}, C)$, *where* $C = \mathcal{A} \times \mathcal{B}$ *and for* $(\alpha, \beta) \in C$, $\alpha \in \mathcal{A}$, $\beta \in \mathcal{B}$, $\mathbb{H}(\alpha, \beta) = \mathbb{F}(\alpha) \cap \mathbb{G}(\beta)$. *We write* $(\mathbb{F}, \mathcal{A}) \,\widetilde{\wedge}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

2.2. Bipolar Soft Sets

Let $\mathcal{U}$ be an initial universe and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of $\mathcal{U}$ and A, B, C are non-empty subsets of E.

Definition 15 ([11]). *A triple* $(\mathbb{F}, \mathbb{G}, A)$ *is called a bipolar soft set over* $\mathcal{U}$, *where* $\mathbb{F}$ *and* $\mathbb{G}$ *are mappings given by* $\mathbb{F} : A \to \mathcal{P}(\mathcal{U})$ *and* $\mathbb{G} : \neg A \to \mathcal{P}(\mathcal{U})$ *such that* $\mathbb{F}(e) \cap \mathbb{G}(e) = \phi$ *for all* $e \in A$.

In other words, a bipolar soft set over $\mathcal{U}$ gives two parameterized families of subsets of the universe $\mathcal{U}$ and the conditions $\mathbb{F}(e) \cap \mathbb{G}(e) = \phi$ for all $e \in A$ are imposed as a consistency constraint.

Definition 16 ([11]). *For two bipolar soft sets* $(\mathbb{F}_1, \mathbb{G}_1, A)$ *and* $(\mathbb{F}_2, \mathbb{G}_2, B)$ *over a common universe* $\mathcal{U}$, *we say that* $(\mathbb{F}_1, \mathbb{G}_1, A)$ *is a bipolar soft subset of* $(\mathbb{F}_2, \mathbb{G}_2, B)$ *if the following is the case:*

1. $A \subseteq B$ *and;*
2. $\mathbb{F}_1(e) \subseteq \mathbb{F}_2(e)$ *and* $\mathbb{G}_2(\neg e) \subseteq \mathbb{G}_1(\neg e)$ *for all* $e \in A$.

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\widetilde{\widetilde{\subseteq}}\, (\mathbb{F}_2, \mathbb{G}_2, B)$.

$(\mathbb{F}_1, \mathbb{G}_1, A)$ is said to be a bipolar soft superset of $(\mathbb{F}_2, \mathbb{G}_2, B)$ if $(\mathbb{F}_2, \mathbb{G}_2, B)$ is a bipolar soft subset of $(\mathbb{F}_1, \mathbb{G}_1, A)$. We denote it by $(\mathbb{F}_1, \mathbb{G}_1, A) \,\widetilde{\widetilde{\supseteq}}\, (\mathbb{F}_2, \mathbb{G}_2, B)$.

Definition 17 ([11]). *Two bipolar soft sets* $(\mathbb{F}_1, \mathbb{G}_1, A)$ *and* $(\mathbb{F}_2, \mathbb{G}_2, B)$ *over a common universe* $\mathcal{U}$ *are said to be bipolar soft equal if* $(\mathbb{F}_1, \mathbb{G}_1, A)$ *is a bipolar soft subset of* $(\mathbb{F}_2, \mathbb{G}_2, B)$ *and* $(\mathbb{F}_2, \mathbb{G}_2, B)$ *is a bipolar soft subset of* $(\mathbb{F}_1, \mathbb{G}_1, A)$.

Definition 18 ([11]). *The complement of a bipolar soft set* $(\mathbb{F}, \mathbb{G}, A)$ *is denoted by* $(\mathbb{F}, \mathbb{G}, A)^c$ *and is defined by* $(\mathbb{F}, \mathbb{G}, A)^c = (\mathbb{F}^c, \mathbb{G}^c, A)$, *where* $\mathbb{F}^c$ *and* $\mathbb{G}^c$ *are mappings given by* $\mathbb{F}^c(e) = \mathbb{G}(\neg e)$ *and* $\mathbb{G}^c(\neg e) = \mathbb{F}(e)$ *for all* $e \in A$.

Definition 19 ([11]). *A bipolar soft set* $(\mathbb{F}, \mathbb{G}, A)$ *over* $\mathcal{U}$ *is said to be a relative null bipolar soft set denoted by* (Φ, Ψ, A), *if for all* $e \in A$, $\Phi(e) = \phi$ *and* $\Psi(\neg e) = \mathcal{U}$.

The relative null bipolar soft set with respect to the universe set of parameters E is called the absolute null bipolar soft set over $\mathcal{U}$ and is denoted by (Φ, Ψ, E).

Definition 20 ([11])**.** *A bipolar soft set $(\mathbb{F}, \mathbb{G}, A)$ over $\mathcal{U}$ is said to be a relative whole bipolar soft set denoted by (Ψ, Φ, A), if for all $e \in A$, $\Psi(e) = \mathcal{U}$ and $\Phi(\neg e) = \phi$.*

The relative whole bipolar soft set with respect to the universe set of parameters E is called the absolute whole bipolar soft set over $\mathcal{U}$ and is denoted by (Ψ, Φ, E).

Definition 21 ([11])**.** *The union of two bipolar soft sets $(\mathbb{F}_1, \mathbb{G}_1, A)$ and $(\mathbb{F}_2, \mathbb{G}_2, B)$ over a common universe $\mathcal{U}$ is a bipolar soft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = A \cup B$ and the following is the case for all $e \in C$:*

$$\mathbb{H}(e) = \begin{cases} \mathbb{F}_1(e) & \text{if } e \in A \setminus B \\ \mathbb{F}_2(e) & \text{if } e \in B \setminus A \\ \mathbb{F}_1(e) \cup \mathbb{F}_2(e) & \text{if } e \in A \cap B \end{cases}$$

$$\mathbb{I}(\neg e) = \begin{cases} \mathbb{G}_1(\neg e) & \text{if } e \in \neg A \setminus \neg B \\ \mathbb{G}_2(\neg e) & \text{if } e \in \neg B \setminus \neg A \\ \mathbb{G}_1(\neg e) \cap \mathbb{G}_2(\neg e) & \text{if } e \in \neg A \cap \neg B \end{cases}$$

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\tilde{\tilde{\cup}}\, (\mathbb{F}_2, \mathbb{G}_2, B) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 22 ([11])**.** *Extended intersection of two bipolar soft sets $(\mathbb{F}_1, \mathbb{G}_1, A)$ and $(\mathbb{F}_2, \mathbb{G}_2, B)$ over a common universe $\mathcal{U}$ is a bipolar soft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = A \cup B$ and the following is the case for all $e \in C$:*

$$\mathbb{H}(e) = \begin{cases} \mathbb{F}_1(e) & \text{if } e \in A \setminus B \\ \mathbb{F}_2(e) & \text{if } e \in B \setminus A \\ \mathbb{F}_1(e) \cap \mathbb{F}_2(e) & \text{if } e \in A \cap B \end{cases}$$

$$\mathbb{I}(\neg e) = \begin{cases} \mathbb{G}_1(\neg e) & \text{if } e \in \neg A \setminus \neg B \\ \mathbb{G}_2(\neg e) & \text{if } e \in \neg B \setminus \neg A \\ \mathbb{G}_1(\neg e) \cup \mathbb{G}_2(\neg e) & \text{if } e \in \neg A \cap \neg B \end{cases}$$

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\tilde{\tilde{\cap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, B) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 23 ([11])**.** *Restricted union of two bipolar soft sets $(\mathbb{F}_1, \mathbb{G}_1, A)$ and $(\mathbb{F}_2, \mathbb{G}_2, B)$ over a common universe $\mathcal{U}$ is a bipolar soft set $(H, \mathbb{I}, C)$, where $C = A \cap B$ and for all $e \in C$, $\mathbb{H}(e) = \mathbb{F}_1(e) \cup \mathbb{F}_2(e)$ and $\mathbb{I}(\neg e) = \mathbb{G}_1(\neg e) \cap \mathbb{G}_2(\neg e)$.*

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\tilde{\tilde{\cup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, B) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 24 ([11])**.** *The intersection of two bipolar soft sets $(\mathbb{F}_1, \mathbb{G}_1, A)$ and $(\mathbb{F}_2, \mathbb{G}_2, B)$ over a common universe $\mathcal{U}$ is a bipolar soft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = A \cap B$ and for all $e \in C$, $\mathbb{H}(e) = \mathbb{F}_1(e) \cap \mathbb{F}_2(e)$ and $\mathbb{I}(\neg e) = \mathbb{G}_1(\neg e) \cup \mathbb{G}_2(\neg e)$.*

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\tilde{\tilde{\cap}}\, (\mathbb{F}_2, \mathbb{G}_2, B) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 25 ([11])**.** *The OR-operation of two bipolar soft sets $(\mathbb{F}_1, \mathbb{G}_1, A)$ and $(\mathbb{F}_2, \mathbb{G}_2, B)$ over a common universe $\mathcal{U}$ is a bipolar soft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = A \times B$ and for all $(a, b) \in C$, $a \in A$, $b \in B$, $\mathbb{H}(a, b) = \mathbb{F}_1(a) \cup \mathbb{F}_2(b)$ and $\mathbb{I}(\neg a, \neg b) = \mathbb{G}_1(\neg a) \cap \mathbb{G}_2(\neg b)$.*

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\tilde{\tilde{\vee}}\, (\mathbb{F}_2, \mathbb{G}_2, B) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 26 ([11])**.** *The AND-operation of two bipolar soft sets $(\mathbb{F}_1, \mathbb{G}_1, A)$ and $(\mathbb{F}_2, \mathbb{G}_2, B)$ over a common universe $\mathcal{U}$ is a bipolar soft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = A \times B$ and for all $(a, b) \in C$, $a \in A$, $b \in B$, $\mathbb{H}(a, b) = \mathbb{F}_1(a) \cap \mathbb{F}_2(b)$ and $\mathbb{I}(\neg a, \neg b) = \mathbb{G}_1(\neg a) \cup \mathbb{G}_2(\neg b)$.*

We write $(\mathbb{F}_1, \mathbb{G}_1, A) \,\tilde{\tilde{\wedge}}\, (\mathbb{F}_2, \mathbb{G}_2, B) = (\mathbb{H}, \mathbb{I}, C)$.

3. Bipolar Hypersoft Sets

In this section, we present the notion of bipolar hypersoft sets and some of its basic operations.

Definition 27. *A triple* $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ *is called a bipolar hypersoft set over* $\mathcal{U}$*, where* $\mathbb{F}$ *and* $\mathbb{G}$ *are mappings given by* $\mathbb{F} : \mathcal{A} \to \mathcal{P}(\mathcal{U})$ *and* $\mathbb{G} : \neg\mathcal{A} \to \mathcal{P}(\mathcal{U})$ *such that* $\mathbb{F}(\alpha) \cap \mathbb{G}(\neg\alpha) = \phi$ *for all* $\alpha \in \mathcal{A}$.

In other words, a bipolar hypersoft set $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ provides two parameterized families of subsets of the universe $\mathcal{U}$ and the consistency constraint condition $\mathbb{F}(\alpha) \cap \mathbb{G}(\neg\alpha) = \phi$ for all $\alpha \in \mathcal{A}$ is imposed.

Now, we can represent a bipolar hypersoft set $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ as the following form.

$$(\mathbb{F}, \mathbb{G}, \mathcal{A}) = \{(\alpha, \mathbb{F}(\alpha), \mathbb{G}(\neg\alpha)) : \alpha \in \mathcal{A} \text{ and } \mathbb{F}(\alpha) \cap \mathbb{G}(\neg\alpha) = \phi\}.$$

Example 1. *Let* $\mathcal{U} = \{h_1, h_2, h_3, h_4\}$ *be the set of TVs under consideration and let the parameters be the following.*

$Company = E_1 = \{e_1 = company\ I, e_2 = company\ II\}, Size = E_2 = \{e_3 = large, e_4 = small\}$, *and* $Color = E_3 = \{e_5 = white, e_6 = black\}$.

Suppose that the following is the case:

$$A_1 = \{e_1, e_2\}, A_2 = \{e_4\}, A_3 = \{e_5\}.$$

Now suppose that Mr. X wants to buy a television according to the following: company I or company II*, small and white. Define a bipolar hypersoft set* $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ *as follows.*

$$(\mathbb{F}, \mathbb{G}, \mathcal{A}) = \{((e_1, e_4, e_5), \{h_1, h_3\}, \{h_2\}), ((e_2, e_4, e_5), \{h_3, h_4\}, \{h_1, h_2\})\}.$$

It is noted that, although Mr. X thinks that h_1 *and* h_3 *are the televisions of company* I*, small and white,* h_2 *is not, whereas* h_4 *is not parameterized. Similarly, Mr. X thinks that* h_3 *and* h_4 *are the televisions of company* II*, small and white, whereas* h_1 *and* h_2 *are not.*

Definition 28. *For two bipolar hypersoft sets* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *and* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *over a common universe* $\mathcal{U}$*, we say that* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *is a bipolar hypersoft subset of* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *if the following is the case.*

1. $\mathcal{A} \subseteq \mathcal{B}$ *and;*
2. $\mathbb{F}_1(\alpha) \subseteq \mathbb{F}_2(\alpha)$ *and* $\mathbb{G}_2(\neg\alpha) \subseteq \mathbb{G}_1(\neg\alpha)$ *for all* $\alpha \in \mathcal{A}$.

We write $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqsubseteq}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$.

$(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ is said to be a bipolar hypersoft superset of $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ if $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ is a bipolar hypersoft subset of $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$. We denote it by $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqsupseteq}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$.

Definition 29. *Two bipolar hypersoft sets* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *and* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *over a common universe* $\mathcal{U}$ *are said to be bipolar hypersoft equal if* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *is a bipolar hypersoft subset of* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *and* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *is a bipolar hypersoft subset of* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$.

Example 2. *Let* $\mathcal{U} = \{h_1, h_2, h_3, h_4\}$ *as follows.*

$$E_1 = \{e_1, e_2\}, E_2 = \{e_3, e_4\}, E_3 = \{e_5, e_6\},$$
$$A_1 = \{e_1, e_2\}, A_2 = \{e_3\}, A_3 = \{e_5\}$$
$$B_1 = \{e_1, e_2\}, B_2 = \{e_3, e_4\}, B_3 = \{e_5\}$$

Let the bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ be defined by the following.

$(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) = \{((e_1, e_3, e_5), \{h_1, h_3\}, \{h_2, h_4\}), ((e_2, e_3, e_5), \{h_3\}, \{h_1, h_4\})\}$.

$(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = \{((e_1, e_3, e_5), \{h_1, h_3, h_4\}, \{h_2\}), ((e_1, e_4, e_5), \{h_2, h_4\}, \{h_1\}),$
$((e_2, e_3, e_5), \{h_1, h_3\}, \{h_4\}), ((e_2, e_4, e_5), \{h_3, h_4\}, \{h_2\})\}$.

Then, $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \mathrel{\tilde{\tilde{\sqsubseteq}}} (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$.

Definition 30. *The complement of a bipolar hypersoft set $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ is denoted by $(\mathbb{F}, \mathbb{G}, \mathcal{A})^c$ and is defined by $(\mathbb{F}, \mathbb{G}, \mathcal{A})^c = (\mathbb{F}^c, \mathbb{G}^c, \mathcal{A})$, where $\mathbb{F}^c$ and $\mathbb{G}^c$ are mappings given by $\mathbb{F}^c(\alpha) = \mathbb{G}(\neg\alpha)$ and $\mathbb{G}^c(\neg\alpha) = \mathbb{F}(\alpha)$ for all $\alpha \in \mathcal{A}$.*

Definition 31. *A bipolar hypersoft set $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ over $\mathcal{U}$ is said to be a relative null bipolar hypersoft set denoted by $(\Phi, \Psi, \mathcal{A})$, if, for all $\alpha \in \mathcal{A}$, $\Phi(\alpha) = \phi$ and $\Psi(\neg\alpha) = \mathcal{U}$.*

The relative null bipolar hypersoft set with respect to the universe set of parameters $\mathcal{E}$ is called the absolute null bipolar hypersoft set over $\mathcal{U}$ and is denoted by $(\Phi, \Psi, \mathcal{E})$.

Definition 32. *A bipolar hypersoft set $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ over $\mathcal{U}$ is said to be a relative whole bipolar hypersoft set that is denoted by $(\Psi, \Phi, \mathcal{A})$, if, for all $\alpha \in \mathcal{A}$, $\Psi(\alpha) = \mathcal{U}$ and $\Phi(\neg\alpha) = \phi$.*

The relative whole bipolar hypersoft set with respect to the universe set of parameters $\mathcal{E}$ is called the absolute whole bipolar hypersoft set over $\mathcal{U}$ and is denoted by $(\Psi, \Phi, \mathcal{E})$.

Definition 33. *The union of two bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ over a common universe $\mathcal{U}$ is a bipolar hypersoft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = \mathcal{A} \cup \mathcal{B}$ and, for all $\alpha \in C$, the following is the case:*

$$\mathbb{H}(\alpha) = \begin{cases} \mathbb{F}_1(\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$\mathbb{I}(\neg\alpha) = \begin{cases} \mathbb{G}_1(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{A} \setminus \neg\mathcal{B} \\ \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{B} \setminus \neg\mathcal{A} \\ \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{A} \cap \neg\mathcal{B} \end{cases}$$

We write $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \mathrel{\tilde{\tilde{\sqcup}}} (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 34. *The extended intersection of two bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ over a common universe $\mathcal{U}$ is a bipolar hypersoft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = \mathcal{A} \cup \mathcal{B}$ and, for all $\alpha \in C$, the following is the case.*

$$\mathbb{H}(\alpha) = \begin{cases} \mathbb{F}_1(\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{F}_1(\alpha) \cap \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$\mathbb{I}(\neg\alpha) = \begin{cases} \mathbb{G}_1(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{A} \setminus \neg\mathcal{B} \\ \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{B} \setminus \neg\mathcal{A} \\ \mathbb{G}_1(\neg\alpha) \cup \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{A} \cap \neg\mathcal{B} \end{cases}$$

We write $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \mathrel{\tilde{\tilde{\sqcap}}}_\varepsilon (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 35. *The restricted union of two bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ over a common universe $\mathcal{U}$ is a bipolar hypersoft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = \mathcal{A} \cap \mathcal{B}$ and the following is the case for all $\alpha \in C$.*

$$\mathbb{H}(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) \text{ and } \mathbb{I}(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha).$$

We write $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 36. *The intersection of two bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ over a common universe $\mathcal{U}$ is a bipolar hypersoft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = \mathcal{A} \cap \mathcal{B}$ and the following is the case for all $\alpha \in C$.*

$$\mathbb{H}(\alpha) = \mathbb{F}_1(\alpha) \cap \mathbb{F}_2(\alpha) \text{ and } \mathbb{I}(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cup \mathbb{G}_2(\neg\alpha).$$

We write $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}, \mathbb{I}, C)$.

Example 3. *Let $\mathcal{U} = \{h_1, h_2, h_3, h_4\}$ as follows.*

$$\begin{gathered} E_1 = \{e_1, e_2\}, E_2 = \{e_3, e_4\}, E_3 = \{e_5, e_6\}, \\ A_1 = \{e_1, e_2\}, A_2 = \{e_3, e_4\}, A_3 = \{e_5\} \\ B_1 = \{e_1, e_2\}, B_2 = \{e_3\}, B_3 = \{e_5, e_6\} \end{gathered}$$

Let the bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ be defined by the following.

$(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) = \{((e_1, e_3, e_5), \{h_4\}, \{h_1, h_2\}), ((e_1, e_4, e_5), \{h_1, h_3\}, \{h_4\}),$
$((e_2, e_3, e_5), \{h_2, h_3\}, \{h_4\}), ((e_2, e_4, e_5), \{h_3\}, \{h_2, h_4\})\}.$

$(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = \{((e_1, e_3, e_5), \{h_1, h_4\}, \{h_3\}), ((e_1, e_3, e_6), \{h_1, h_4\}, \{h_3\}),$
$((e_2, e_3, e_5), \{h_1\}, \{h_2, h_3, h_4\}), ((e_2, e_3, e_6), \{h_3, h_4\}, \{h_2\})\}$.

Suppose that $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}_1, \mathbb{I}_1, C)$. *Then the following is the case.*

$(\mathbb{H}_1, \mathbb{I}_1, C) = \{((e_1, e_3, e_5), \{h_1, h_4\}, \phi), ((e_1, e_4, e_5), \{h_1, h_3\}, \{h_4\}),$
$((e_2, e_3, e_5), \{h_1, h_2, h_3\}, \{h_4\}), ((e_2, e_4, e_5), \{h_3\}, \{h_2, h_4\}),$
$((e_1, e_3, e_6), \{h_1, h_4\}, \{h_3\}), ((e_2, e_3, e_6), \{h_3, h_4\}, \{h_2\})\}.$

Moreover, let $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}_2, \mathbb{I}_2, C)$. *Then the following is the case.*

$(\mathbb{H}_2, \mathbb{I}_2, C) = \{((e_1, e_3, e_5), \{h_4\}, \{h_1, h_2, h_3\}), ((e_1, e_4, e_5), \{h_1, h_3\}, \{h_4\}),$
$((e_2, e_3, e_5), \phi, \{h_2, h_3, h_4\}), ((e_2, e_4, e_5), \{h_3\}, \{h_2, h_4\}),$
$((e_1, e_3, e_6), \{h_1, h_4\}, \{h_3\}), ((e_2, e_3, e_6), \{h_3, h_4\}, \{h_2\})\}$.

Definition 37. *The difference of two bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ over a common universe $\mathcal{U}$ is a bipolar hypersoft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = \mathcal{A} \cap \mathcal{B}$ and the following is the case for all $\alpha \in C$.*

$$\begin{gathered} \mathbb{H}(\alpha) = \mathbb{F}_1(\alpha) \cap {\mathbb{F}_2}^c(\alpha) = \mathbb{F}_1(\alpha) \cap \mathbb{G}_2(\neg\alpha) \text{ and} \\ \mathbb{I}(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cup {\mathbb{G}_2}^c(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cup \mathbb{F}_2(\alpha). \end{gathered}$$

We write $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \setminus (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}$
$({\mathbb{F}_2}^c, {\mathbb{G}_2}^c,$
$\mathcal{B}) = (\mathbb{H}, \mathbb{I}, C)$.

Definition 38. *The OR-operation of two bipolar hypersoft sets $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ over a common universe $\mathcal{U}$ is a bipolar hypersoft set $(\mathbb{H}, \mathbb{I}, C)$, where $C = \mathcal{A} \times \mathcal{B}$ and the following is the case for all $(\alpha, \beta) \in C$, $\alpha \in \mathcal{A}$, $\beta \in \mathcal{B}$.*

$$\mathbb{H}(\alpha,\beta)=\mathbb{F}_1(\alpha)\cup\mathbb{F}_2(\beta) \text{ and } \mathbb{I}(\neg\alpha,\neg\beta)=\mathbb{G}_1(\neg\alpha)\cap\mathbb{G}_2(\neg\beta).$$

We write $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\vee}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{B})=(\mathbb{H},\mathbb{I},C)$.

Definition 39. *The AND-operation of two bipolar hypersoft sets* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$ *and* $(\mathbb{F}_2,\mathbb{G}_2,\mathcal{B})$ *over a common universe* $\mathcal{U}$ *is a bipolar hypersoft set* $(\mathbb{H},\mathbb{I},C)$, *where* $C=\mathcal{A}\times\mathcal{B}$ *and the following is the case for all* $(\alpha,\beta)\in C$. $\alpha\in\mathcal{A}$, $\beta\in\mathcal{B}$,

$$\mathbb{H}(\alpha,\beta)=\mathbb{F}_1(\alpha)\cap\mathbb{F}_2(\beta) \text{ and } \mathbb{I}(\neg\alpha,\neg\beta)=\mathbb{G}_1(\neg\alpha)\cup\mathbb{G}_2(\neg\beta).$$

We write $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\wedge}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{B})=(\mathbb{H},\mathbb{I},C)$.

Example 4. *Consider Example 3, let* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\vee}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{B})=(\mathbb{H}_1,\mathbb{I}_1,C)$, *then the following is the case.*

$(\mathbb{H}_1,\mathbb{I}_1,C)=\{((e_1,e_3,e_5),(e_1,e_3,e_5),\{h_1,h_4\},\phi),((e_1,e_3,e_5),(e_1,e_3,e_6),\{h_1,h_4\},\phi),$
$((e_1,e_3,e_5),(e_2,e_3,e_5),\{h_1,h_4\},\{h_2\}),((e_1,e_3,e_5),(e_2,e_3,e_6),\{h_3,h_4\},\{h_2\}),$
$((e_1,e_4,e_5),(e_1,e_3,e_5),\{h_1,h_3,h_4\},\phi),((e_1,e_4,e_5),(e_1,e_3,e_6),\{h_1,h_3,h_4\},\phi),$
$((e_1,e_4,e_5),(e_2,e_3,e_5),\{h_1,h_3\},\{h_4\}),((e_1,e_4,e_5),(e_2,e_3,e_6),\{h_1,h_3,h_4\},\phi),$
$((e_2,e_3,e_5),(e_1,e_3,e_5),\mathcal{U},\phi),((e_2,e_3,e_5),(e_1,e_3,e_6),\mathcal{U},\phi),$
$((e_2,e_3,e_5),(e_2,e_3,e_5),\{h_1,h_2,h_3\},\{h_4\}),((e_2,e_3,e_5),(e_2,e_3,e_6),\{h_2,h_3,h_4\},\phi),$
$((e_2,e_4,e_5),(e_1,e_3,e_5),\{h_1,h_3,h_4\},\phi),((e_2,e_4,e_5),(e_1,e_3,e_6),\{h_1,h_3,h_4\},\phi),$
$((e_2,e_4,e_5),(e_2,e_3,e_5),\{h_1,h_3\},\{h_2,h_4\}),((e_2,e_4,e_5),(e_2,e_3,e_6),\{h_3,h_4\},\{h_2\})\}.$

Now, if we let $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\wedge}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{B})=(\mathbb{H}_2,\mathbb{I}_2,C)$, *then the following is the case.*

$(\mathbb{H}_2,\mathbb{I}_2,C)=\{((e_1,e_3,e_5),(e_1,e_3,e_5),\{h_4\},\{h_1,h_2,h_3\}),((e_1,e_3,e_5),(e_1,e_3,e_6),\{h_4\},\{h_1,h_2,h_3\}),$
$((e_1,e_3,e_5),(e_2,e_3,e_5),\phi,\mathcal{U}),((e_1,e_3,e_5),(e_2,e_3,e_6),\{h_4\},\{h_1,h_2\}),$
$((e_1,e_4,e_5),(e_1,e_3,e_5),\{h_1\},\{h_3,h_4\}),((e_1,e_4,e_5),(e_1,e_3,e_6),\{h_1\},\{h_3,h_4\}),$
$((e_1,e_4,e_5),(e_2,e_3,e_5),\{h_1\},\{h_2,h_3,h_4\}),((e_1,e_4,e_5),(e_2,e_3,e_6),\{h_3\},\{h_2,h_4\}),$
$((e_2,e_3,e_5),(e_1,e_3,e_5),\phi,\{h_3,h_4\}),((e_2,e_3,e_5),(e_1,e_3,e_6),\phi,\{h_3,h_4\}),$
$((e_2,e_3,e_5),(e_2,e_3,e_5),\phi,\{h_2,h_3,h_4\}),((e_2,e_3,e_5),(e_2,e_3,e_6),\{h_3\},\{h_2,h_4\}),$
$((e_2,e_4,e_5),(e_1,e_3,e_5),\phi,\{h_2,h_3,h_4\}),((e_2,e_4,e_5),(e_1,e_3,e_6),\phi,\{h_2,h_3,h_4\}),$
$((e_2,e_4,e_5),(e_2,e_3,e_5),\phi,\{h_2,h_3,h_4\}),((e_2,e_4,e_5),(e_2,e_3,e_6),\{h_3\},\{h_2,h_4\})\}.$

Proposition 2. *If* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$, $(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})$ *and* $(\mathbb{F}_3,\mathbb{G}_3,\mathcal{A})$ *are any bipolar hypersoft sets over a universe* $\mathcal{U}$. *Then the following is the case:*

1. $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$;
2. $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\Psi,\Phi,\mathcal{A})$;
3. $(\Phi,\Psi,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$;
4. *If* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})$ *and* $(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_3,\mathbb{G}_3,\mathcal{A})$ *then* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_3,\mathbb{G}_3,\mathcal{A})$;
5. *If* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})=(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})$ *and* $(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})=(\mathbb{F}_3,\mathbb{G}_3,\mathcal{A})$ *then* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})=(\mathbb{F}_3,\mathbb{G}_3,\mathcal{A})$.

Proof. This is straightforward. □

Proposition 3. *Let* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$ *and* $(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})$ *be two bipolar hypersoft sets over a universe* $\mathcal{U}$. *Then the following is the case:*

1. $(\Phi,\Psi,\mathcal{A})^c=(\Psi,\Phi,\mathcal{A})$ *and* $(\Psi,\Phi,\mathcal{A})^c=(\Phi,\Psi,\mathcal{A})$;
2. $((\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})^c)^c=(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$;
3. $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})$ *then* $(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})^c\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})^c$;
4. $(\Phi,\Psi,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqcap}}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})^c\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqcup}}_{\Re}\,(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})^c\,\tilde{\tilde{\sqsubseteq}}\,(\Psi,\Phi,\mathcal{A})$;
5. *If* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqsubseteq}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})$ *then* $(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})\,\tilde{\tilde{\sqcap}}\,(\mathbb{F}_2,\mathbb{G}_2,\mathcal{A})=(\mathbb{F}_1,\mathbb{G}_1,\mathcal{A})$;

6. *If* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqsubseteq}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A})$ *then* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A}) = (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A})$.

Proof. This is straightforward. □

Proposition 4. *If* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *and* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *are two bipolar hypersoft sets over a universe* $\mathcal{U}$*. Then the following is the case:*

1. $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c$;
2. $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c$;
3. $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c$;
4. $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c$;
5. $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\vee}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\wedge}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c$;
6. $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\wedge}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\vee}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c$.

Proof. (1) Let $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}_1, \mathbb{I}_1, C)$ where $C = \mathcal{A} \cup \mathcal{B}$. Then $((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}))^c = (\mathbb{H}_1, \mathbb{I}_1, C)^c = (\mathbb{H}_1{}^c, \mathbb{I}_1{}^c, C)$. By definition, the following is the case.

$$\mathbb{H}_1(\alpha) = \begin{cases} \mathbb{F}_1(\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$\mathbb{I}_1(\neg\alpha) = \begin{cases} \mathbb{G}_1(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{A} \setminus \neg\mathcal{B} \\ \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{B} \setminus \neg\mathcal{A} \\ \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{A} \cap \neg\mathcal{B} \end{cases}$$

Thus, we have the following.

$$\mathbb{H}_1{}^c(\alpha) = \mathbb{I}_1(\neg\alpha) = \begin{cases} \mathbb{G}_1(\neg\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$\mathbb{I}_1{}^c(\neg\alpha) = \mathbb{H}_1(\alpha) = \begin{cases} \mathbb{F}_1(\alpha) & \text{if } \alpha \in \neg\mathcal{A} \setminus \neg\mathcal{B} \\ \mathbb{F}_2(\alpha) & \text{if } \alpha \in \neg\mathcal{B} \setminus \neg\mathcal{A} \\ \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) & \text{if } \alpha \in \neg\mathcal{A} \cap \neg\mathcal{B} \end{cases}$$

Moreover, let $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})^c \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})^c = (\mathbb{F}_1{}^c, \mathbb{G}_1{}^c, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2{}^c, \mathbb{G}_2{}^c, \mathcal{B})^c = (\mathbb{H}_2, \mathbb{I}_2, C)$ where $C = \mathcal{A} \cup \mathcal{B}$, then the following is obtained.

$$\mathbb{H}_2(\alpha) = \begin{cases} \mathbb{F}_1{}^c(\alpha) = \mathbb{G}_1(\neg\alpha) & \text{if } \alpha \in \mathcal{A} \setminus \mathcal{B} \\ \mathbb{F}_2{}^c(\alpha) = \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{A} \\ \mathbb{F}_1{}^c(\alpha) \cap \mathbb{F}_2{}^c(\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \mathcal{A} \cap \mathcal{B} \end{cases}$$

$$\mathbb{I}_2(\neg\alpha) = \begin{cases} \mathbb{G}_1{}^c(\neg\alpha) = \mathbb{F}_1(\alpha) & \text{if } \alpha \in \neg\mathcal{A} \setminus \neg\mathcal{B} \\ \mathbb{G}_2{}^c(\neg\alpha) = \mathbb{F}_2(\alpha) & \text{if } \alpha \in \neg\mathcal{B} \setminus \neg\mathcal{A} \\ \mathbb{G}_1{}^c(\neg\alpha) \cup \mathbb{G}_2{}^c(\neg\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) & \text{if } \alpha \in \neg\mathcal{A} \cap \neg\mathcal{B} \end{cases}$$

Since $(\mathbb{H}_1{}^c, \mathbb{I}_1{}^c, C)$ and $(\mathbb{H}_2, \mathbb{I}_2, C)$ are the same set-valued mapping for all $\alpha \in C$, the proof is completed.

The remaining parts can be proved with the same method. □

Proposition 5. *If* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *and* $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{A})$ *are two bipolar hypersoft sets over a universe* $\mathcal{U}$*, then we have the following:*

1. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A})$;
2. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{A})$;

3. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *and* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$;
4. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\Phi, \Psi, \mathcal{A}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$ *and* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\Phi, \Psi, \mathcal{A}) = (\Phi, \Psi, \mathcal{A})$;
5. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\Psi, \Phi, \mathcal{A}) = (\Psi, \Phi, \mathcal{A})$ *and* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\Psi, \Phi, \mathcal{A}) = (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$.

Proof. This is straightforward. □

Proposition 6. *Let* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$, $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *and* $(\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})$ *be any bipolar hypersoft sets over a universe* $\mathcal{U}$*, then we have the following:*

1. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$;
2. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$;
3. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$;
4. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$;
5. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})$;
6. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})$;
7. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})$;
8. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})$.

Proof. (7) Suppose that $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}) = (\mathbb{H}_1, \mathbb{I}_1, \mathcal{B} \cap \mathcal{C})$. Then, for all $\alpha \in \mathcal{B} \cap \mathcal{C}$, we have the following.

$$\mathbb{H}_1(\alpha) = \mathbb{F}_2(\alpha) \cup \mathbb{F}_3(\alpha) \text{ and } \mathbb{I}_1(\neg\alpha) = \mathbb{G}_2(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha).$$

Assume $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{H}_1, \mathbb{I}_1, \mathcal{B} \cap \mathcal{C}) = (\mathbb{H}_2, \mathbb{I}_2, \mathcal{A} \cap (\mathcal{B} \cap \mathcal{C}))$. Then, for all $\alpha \in \mathcal{A} \cap (\mathcal{B} \cap \mathcal{C})$, we have the following:

$$\begin{gathered}\mathbb{H}_2(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{H}_1(\alpha) = \mathbb{F}_1(\alpha) \cup (\mathbb{F}_2(\alpha) \cup \mathbb{F}_3(\alpha)) \text{ and} \\ \mathbb{I}_2(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{I}_1(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap (\mathbb{G}_2(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha)).\end{gathered}$$

On the other hand, let $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}_3, \mathbb{I}_3, \mathcal{A} \cap \mathcal{B})$. Then, for all $\alpha \in \mathcal{A} \cap \mathcal{B}$, we have the following.

$$\mathbb{H}_3(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) \text{ and } \mathbb{I}_3(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha).$$

Assume $(\mathbb{H}_3, \mathbb{I}_3, \mathcal{A} \cap \mathcal{B}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}) = (\mathbb{H}_4, \mathbb{I}_4, (\mathcal{A} \cap \mathcal{B}) \cap \mathcal{C})$. Then, for all $\alpha \in (\mathcal{A} \cap \mathcal{B}) \cap \mathcal{C}$, we have the following.

$$\begin{gathered}\mathbb{H}_4(\alpha) = \mathbb{H}_3(\alpha) \cup \mathbb{F}_3(\alpha) = (\mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha)) \cup \mathbb{F}_3(\alpha) \text{ and} \\ \mathbb{I}_4(\neg\alpha) = \mathbb{I}_3(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha) = (\mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha)) \cap \mathbb{G}_3(\neg\alpha).\end{gathered}$$

Since $(\mathbb{H}_2, \mathbb{I}_2, \mathcal{A} \cap (\mathcal{B} \cap \mathcal{C}))$ and $(\mathbb{H}_4, \mathbb{I}_4, (\mathcal{A} \cap \mathcal{B}) \cap \mathcal{C})$ are the same set-valued mapping for all $\alpha \in \mathcal{A} \cap (\mathcal{B} \cap \mathcal{C}) = (\mathcal{A} \cap \mathcal{B}) \cap \mathcal{C}$, the proof is completed.

The remaining parts can be proved with the same method. □

Proposition 7. *Let* $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A})$, $(\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})$ *and* $(\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})$ *be any bipolar hypersoft sets over a universe* $\mathcal{U}$*. Then, the following is the case:*

1. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcap}}\, ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}))$;
2. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcup}}\, ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}))$;
3. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}))$;
4. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\tilde{\tilde{\sqcap}}_{\varepsilon}\, ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\tilde{\tilde{\sqcup}}_{\mathfrak{R}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}))$;

5. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\widetilde{\widetilde{\sqcap}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\widetilde{\widetilde{\sqcap}}\, ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}))$;
6. $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcap}}\, ((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcap}}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B})) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, ((\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcap}}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}))$.

Proof. (4) Suppose that $((\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) \,\widetilde{\widetilde{\sqcap}}_{\varepsilon}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C})) = (\mathbb{H}_1, \mathbb{I}_1, \mathcal{B} \cup \mathcal{C})$. For all $\alpha \in \mathcal{B} \cup \mathcal{C}$, we have the following.

$$\mathbb{H}_1(\alpha) = \begin{cases} \mathbb{F}_2(\alpha) & \text{if } \alpha \in \mathcal{B} \setminus \mathcal{C} \\ \mathbb{F}_3(\alpha) & \text{if } \alpha \in \mathcal{C} \setminus \mathcal{B} \\ \mathbb{F}_2(\alpha) \cap \mathbb{F}_3(\alpha) & \text{if } \alpha \in \mathcal{B} \cap \mathcal{C} \end{cases}$$

$$\mathbb{I}_1(\neg\alpha) = \begin{cases} \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{B} \setminus \neg\mathcal{C} \\ \mathbb{G}_3(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{C} \setminus \neg\mathcal{B} \\ \mathbb{G}_2(\neg\alpha) \cup \mathbb{G}_3(\neg\alpha) & \text{if } \alpha \in \neg\mathcal{B} \cap \neg\mathcal{C} \end{cases}$$

Assume that $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, (\mathbb{H}_1, \mathbb{I}_1, \mathcal{B} \cup \mathcal{C}) = (\mathbb{H}_2, \mathbb{I}_2, \mathcal{A} \cap (\mathcal{B} \cup \mathcal{C})) = (\mathbb{H}_2, \mathbb{I}_2, M \cup N)$ where $M = \mathcal{A} \cap \mathcal{B}$ and $N = \mathcal{A} \cap \mathcal{C}$. For all $\alpha \in M \cup N$, we have the following.

$$\mathbb{H}_2(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{H}_1(\alpha), \text{ and } \mathbb{I}_2(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{I}_1(\neg\alpha).$$

In this case, we obtain the following.

$$\mathbb{H}_2(\alpha) = \begin{cases} \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) & \text{if } \alpha \in M \setminus N \\ \mathbb{F}_1(\alpha) \cup \mathbb{F}_3(\alpha) & \text{if } \alpha \in N \setminus M \\ \mathbb{F}_1(\alpha) \cup (\mathbb{F}_2(\alpha) \cap \mathbb{F}_3(\alpha)) & \text{if } \alpha \in M \cap N \end{cases}$$

$$\mathbb{I}_2(\neg\alpha) = \begin{cases} \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg M \setminus \neg N \\ \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha) & \text{if } \alpha \in \neg N \setminus \neg M \\ \mathbb{G}_1(\neg\alpha) \cap (\mathbb{G}_2(\neg\alpha) \cup \mathbb{G}_3(\neg\alpha)) & \text{if } \alpha \in \neg M \cap \neg N \end{cases}$$

On the other hand, let $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, (\mathbb{F}_2, \mathbb{G}_2, \mathcal{B}) = (\mathbb{H}_3, \mathbb{I}_3, \mathcal{A} \cap \mathcal{B})$. For all $\alpha \in \mathcal{A} \cap \mathcal{B}$, we have the following.

$$\mathbb{H}_3(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) \text{ and } \mathbb{I}_3(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha)$$

Let $(\mathbb{F}_1, \mathbb{G}_1, \mathcal{A}) \,\widetilde{\widetilde{\sqcup}}_{\Re}\, (\mathbb{F}_3, \mathbb{G}_3, \mathcal{C}) = (\mathbb{H}_4, \mathbb{I}_4, \mathcal{A} \cap \mathcal{C})$. For all $\alpha \in \mathcal{A} \cap \mathcal{C}$, we obtain the following.

$$\mathbb{H}_4(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_3(\alpha) \text{ and } \mathbb{I}_3(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha).$$

Now, suppose that $(\mathbb{H}_3, \mathbb{I}_3, \mathcal{A} \cap \mathcal{B}) \,\widetilde{\widetilde{\sqcap}}_{\varepsilon}\, (\mathbb{H}_4, \mathbb{I}_4, \mathcal{A} \cap \mathcal{C}) = (\mathbb{H}_5, \mathbb{I}_5, M \cup N)$ where $M = \mathcal{A} \cap \mathcal{B}$ and $N = \mathcal{A} \cap \mathcal{C}$. For all $\alpha \in M \cup N$, we have the following.

$$\mathbb{H}_5(\alpha) = \begin{cases} \mathbb{H}_3(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha) & \text{if } \alpha \in M \setminus N \\ \mathbb{H}_4(\alpha) = \mathbb{F}_1(\alpha) \cup \mathbb{F}_3(\alpha) & \text{if } \alpha \in N \setminus M \\ \mathbb{H}_3(\alpha) \cap \mathbb{H}_4(\alpha) = (\mathbb{F}_1(\alpha) \cup \mathbb{F}_2(\alpha)) \cap (\mathbb{F}_1(\alpha) \cup \mathbb{F}_3(\alpha)) & \text{if } \alpha \in M \cap N \end{cases}$$

$$\mathbb{I}_5(\neg\alpha) = \begin{cases} \mathbb{I}_3(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha) & \text{if } \alpha \in \neg M \setminus \neg N \\ \mathbb{I}_4(\neg\alpha) = \mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha) & \text{if } \alpha \in \neg N \setminus \neg M \\ \mathbb{I}_3(\neg\alpha) \cup \mathbb{I}_4(\neg\alpha) = (\mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_2(\neg\alpha)) \cup (\mathbb{G}_1(\neg\alpha) \cap \mathbb{G}_3(\neg\alpha)) & \text{if } \alpha \in \neg M \cap \neg N \end{cases}$$

Since $(\mathbb{H}_2, \mathbb{I}_2, M \cup N)$ and $(\mathbb{H}_5, \mathbb{I}_5, M \cup N)$ are the same set-valued mapping for all $\alpha \in M \cup N$, the proof is completed.

The remaining parts can be proved with the same method. □

A representation of a bipolar hypersoft set in the form of a matrix or a table may be desired for storing it in a computer. The $(i,j)th$ entry in table is as follows.

$$a^{\mathbb{F}}_{i,j} = \begin{cases} 1 & \text{if } h_i \in \mathbb{F}(\alpha_j) \\ 0 & \text{otherwise} \end{cases}, \qquad a^{\mathbb{G}}_{i,j} = \begin{cases} 1 & \text{if } h_i \in \mathbb{G}(\neg\alpha_j) \\ 0 & \text{otherwise} \end{cases}$$

With reference to Example 1, the tabular representation of the bipolar hypersoft set for each of the functions $\mathbb{F}$ and $\mathbb{G}$ is given in Table 1.

Table 1. A pair of tables are used to represent $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ in a tabular format.

$\mathbb{F}$	(e_1, e_4, e_5)	(e_2, e_4, e_5)
h_1	1	0
h_2	0	0
h_3	1	1
h_4	0	1
$\mathbb{G}$	$\neg(e_1, e_4, e_5)$	$\neg(e_2, e_4, e_5)$
h_1	0	1
h_2	1	1
h_3	0	0
h_4	0	0

Moreover, we can represent a bipolar hypersoft with the help of a single table by inputting the following.

$$a_{i,j} = \begin{cases} 1 & \text{if } h_i \in \mathbb{F}(\alpha_j) \\ 0 & \text{if } h_i \in \mathcal{U} \setminus \{\mathbb{F}(\alpha_j) \cup \mathbb{G}(\neg\alpha_j)\} \\ -1 & \text{if } h_i \in \mathbb{G}(\neg\alpha_j) \end{cases}$$

With reference to Example 1, the tabular representation of the bipolar hypersoft set is given in Table 2.

Table 2. A single table is used to represent $(\mathbb{F}, \mathbb{G}, \mathcal{A})$ in a tabular format.

$(\mathbb{F}, \mathbb{G}, \mathcal{A})$	(e_1, e_4, e_5)	(e_2, e_4, e_5)
h_1	1	-1
h_2	-1	-1
h_3	1	1
h_4	0	1

4. Conclusions

Hypersoft sets are derived by transforming the approximate function in the structure of a soft set into a multi-attribute approximate function. Meanwhile, bipolarity refers to an explicit handling of positive and negative sides of information. In this paper, we have introduced the concept of bipolar hypersoft sets with some basic definitions. After that, we proposed some operations on the bipolar hypersoft sets such as subset, complement, difference, extended intersection, restricted union, intersection, union, AND and OR. Finally, the necessity and possibility operations on bipolar hypersoft sets with suitable examples and properties have been presented. For future trends, we can construct the bipolar hypersoft points and operations on bipolar hypersoft functions. Furthermore, we can define the bipolar hypersoft topological spaces with their properties by using the

proposed operations. Finally, we provided an application of the bipolar hypersoft set in a decision-making problem.

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